Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

1. Find, using calculus, the x coordinate of the stationary point on the curve with equation

$$y = (2x + 5)e^{3x}$$

(4)

1. At a stationary point, $\frac{dy}{dx} = 0$

$$\boxed{\text{PRODUCT RULE}: \quad y = uv \quad y' = u'v + uv'}$$

$y = (2x+5)e^{3x}$

$u = 2x + 5 \qquad \frac{du}{dx} = 2$

$v = e^{3x} \qquad \frac{dv}{dx} = 3e^{3x}$

$\frac{dy}{dx} = 2(e^{3x}) + (2x+5)(3e^{3x})$

$= e^{3x}(2 + 6x + 15)$

$= (6x+17)e^{3x}$

At stationary point (x, y) $\rightarrow$ $(6x+17)e^{3x} = 0$

$e^{3x} \neq 0$ for $x \in R$

$\therefore \quad 6x + 17 = 0$

$x = -\frac{17}{6}$

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2. (a) Show that the equation

$$8\cos\theta = 3\operatorname{cosec}\theta$$

can be written in the form

$$\sin 2\theta = k$$

where k is a constant to be found.

(3)

(b) Hence find the smallest positive solution of the equation

$$8\cos\theta = 3\operatorname{cosec}\theta$$

giving your answer, in degrees, to one decimal place.

(2)

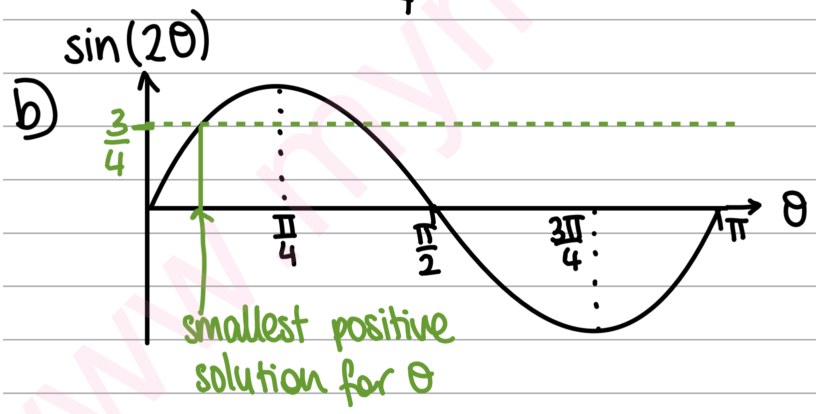
2. a) $8\cos(\theta) = \dfrac{3}{\sin(\theta)}$

$8\cos(\theta)\sin(\theta) = 3$

$4(2\cos(\theta)\sin(\theta)) = 3$ ← $\boxed{\sin(2A) = 2\sin(A)\cos(A)}$

$\sin(2\theta) = \dfrac{3}{4}$

b)



$\sin(2\theta) = \dfrac{3}{4}$ $\qquad \theta = 24.3°$

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3. (i) Find, in simplest form,

$$\int (2x-5)^7 \, dx$$

(2)

(ii) Show, by algebraic integration, that

$$\int_0^{\frac{\pi}{3}} \frac{4\sin x}{1+2\cos x} \, dx = \ln a$$

where a is a rational constant to be found.

(4)

3. (i) $\int (2x-5)^7 \, dx$

$u = 2x-5 \qquad \frac{du}{dx} = 2 \rightarrow dx = \frac{du}{2}$

$\int u^7 \times \frac{du}{2}$

$= \int \frac{u^7}{2} \, du$

$= \frac{u^8}{16} + c \qquad = \frac{(2x-5)^8}{16} + c$

(ii) $\int_0^{\frac{\pi}{3}} \frac{4\sin(x)}{1+2\cos(x)} \, dx$

$u = 1+2\cos(x) \qquad \frac{du}{dx} = -2\sin(x) \rightarrow dx = \frac{du}{-2\sin(x)}$

$\downarrow$

change limits as well

$\frac{\pi}{3} \rightarrow 1+2\cos\left(\frac{\pi}{3}\right) = 2$

$0 \rightarrow 1+2\cos(0) = 3$

Question 3 continued

$$\therefore \int_3^2 \frac{4\cancel{\sin(x)}}{u} \frac{du}{-2\cancel{\sin(x)}}$$

$$= \int_3^2 \frac{-2}{u}\, du$$

$$= \left[-2\ln(u)\right]_3^2 = -2\ln(2) + 2\ln(3)$$

$$= 2\ln\left(\frac{3}{2}\right) = \ln\left(\frac{9}{4}\right)$$

$$\therefore a = \frac{9}{4}$$

Q3

(Total 6 marks)

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4. The growth of a weed on the surface of a pond is being studied.

The surface area of the pond covered by the weed, $A\,\text{m}^2$, is modelled by the equation

$$A = \frac{80pe^{0.15t}}{pe^{0.15t} + 4}$$

where p is a positive constant and t is the number of days after the start of the study.

Given that

- $30\,\text{m}^2$ of the surface of the pond was covered by the weed at the start of the study
- $50\,\text{m}^2$ of the surface of the pond was covered by the weed T days after the start of the study

(a) show that $p = 2.4$

(2)

(b) find the value of T, giving your answer to one decimal place.

(*Solutions relying entirely on graphical or numerical methods are not acceptable.*)

(4)

The weed grows until it covers the surface of the pond.

(c) Find, according to the model, the maximum possible surface area of the pond.

(1)

4.a) $A = \dfrac{80pe^{0.15t}}{pe^{0.15t} + 4}$

when $t = 0 \rightarrow A = 30$

$30 = \dfrac{80pe^{0.15(0)}}{pe^{0.15(0)} + 4} = \dfrac{80p}{p+4}$

$30p + 120 = 80p$

$p = \dfrac{120}{50} = 2.4$

b) when $t = T \rightarrow A = 50$

P 6 9 3 1 1 A 0 8 3 2

Question 4 continued

$$50 = \frac{80(2.4)e^{0.15(T)}}{(2.4)e^{0.15(T)} + 4}$$

$$50\left(2.4e^{0.15T} + 4\right) = 192e^{0.15T}$$

$$120e^{0.15T} + 200 = 192e^{0.15T}$$

$$e^{0.15T} = \frac{25}{9}$$

$$T = \frac{20}{3}\ln\left(\frac{25}{9}\right) = 6.8$$

c) $$A = \frac{192e^{0.15t}}{2.4e^{0.15t} + 4}$$

$\leftarrow$ divide through by $e^{0.15t}$

$$= \frac{192}{2.4 + \cancel{\frac{4}{e^{0.15t}}}^{\to 0}}$$

as $t \to \infty$

$e^{0.15t} \to \infty$

$\therefore \frac{4}{e^{0.15t}} \to 0$

$$\therefore A \to \frac{192}{2.4}$$

$$A \to 80\text{m}^2$$

Q4

(Total 7 marks)

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5.

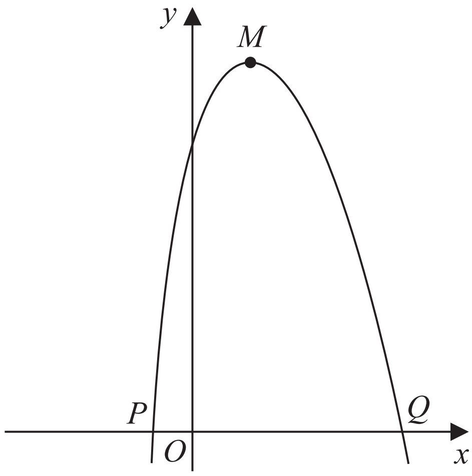


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = 6\ln(2x+3) - \frac{1}{2}x^2 + 4 \qquad x > -\frac{3}{2}$$

The curve cuts the negative x-axis at the point P, as shown in Figure 1.

(a) Show that the x coordinate of P lies in the interval $[-1.25, -1.2]$

(2)

The curve cuts the positive x-axis at the point Q, also shown in Figure 1.

Using the iterative formula

$$x_{n+1} = \sqrt{12\ln(2x_n + 3) + 8} \quad \text{with} \quad x_1 = 6$$

(b) (i) find, to 4 decimal places, the value of x_2

(ii) find, by continued iteration, the x coordinate of Q. Give your answer to 4 decimal places.

(3)

The curve has a maximum turning point at M, as shown in Figure 1.

(c) Using calculus and showing each stage of your working, find the x coordinate of M.

(4)

5.ⓐ $y(-1.25) = -0.9$
$y(-1.2) = 0.2$
} because there is a sign change, & given that the graph is continuous between these points, the graph must cross the x-axis between these points

$\therefore$ P lies in $[-1.25, -1.2]$

Question 5 continued

b) (i) $x_{n+1} = \sqrt{12\ln(2x_n+3)+8}$

$x_1 = 6$

$x_2 = x_{1+1} = \sqrt{12\ln(2(6)+3)+8} = 6.3637$

$x_3 = 6.40819$

$x_4 = 6.41347$

$x_5 = 6.41409$

$x_6 = 6.41416$

$x_7 = 6.41417$

both round to 6.4142 (2 d.p.)

$\therefore x = 6.4142$

c) $y = 6\ln(2x+3) - \frac{x^2}{2} + 4$

At max (M) $\rightarrow \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{6(2)}{2x+3} - \frac{\cancel{2}x}{\cancel{2}}$

$= \frac{12}{2x+3} - x$

At M $\frac{12}{2x+3} - x = 0$

$12 = (2x+3)x$

$2x^2 + 3x - 12 = 0$

$x = \frac{-3 \pm \sqrt{105}}{4}$

reject as $x > -\frac{3}{2}$

$\therefore x = \frac{-3+\sqrt{105}}{4}$

6. The function f is defined by

$$f(x) = \frac{5x-3}{x-4} \qquad x > 4$$

(a) Show, by using calculus, that f is a decreasing function.
(3)

(b) Find f^{-1}
(3)

(c) (i) Show that $ff(x) = \dfrac{ax+b}{x+c}$ where a, b and c are constants to be found.

(ii) Deduce the range of ff.
(5)

6. a) $f(x) = \dfrac{5x-3}{x-4}$

Quotient rule for differentiating : $y = \dfrac{u}{v} \rightarrow \dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$u = 5x-3 \qquad \dfrac{du}{dx} = 5$

$v = x-4 \qquad \dfrac{dv}{dx} = 1$

$f'(x) = \dfrac{(x-4)(5)-(5x-3)(1)}{(x-4)^2} = \dfrac{-17}{(x-4)^2}$ ∴ negative

always positive for $x \in \mathbb{R}$

because $f'(x)$ always < 0

$f(x)$ is a decreasing function

b) to find $f^{-1}(x)$: $f(x) = \dfrac{5x-3}{x-4}$

① write the function using a "y" and set equal to "x" : $x = \dfrac{5y-3}{y+4}$

② rearrange to make y the subject : $xy + 4x = 5y - 3$

Question 6 continued

$$y = \frac{4x+3}{5-x}$$

③ replace y with $f^{-1}(x)$:

$$f^{-1}(x) = \frac{4x+3}{5-x}$$

Because we are told to find $f^{-1}(x)$, we must also state the domain of the inverse function :

↑ domain refers to the set of values we are allowed to plug into our function

domain of inverse function = range of function

↑ range refers to all possible values of a function

∴ domain of $f^{-1}(x)$ = range of $f(x)$

$f(x) = \frac{5x-3}{x-4}$ $x > 4$,

as $x \to \infty$ $f(x) = \frac{5 - \frac{3}{x} \to 0}{1 - \frac{4}{x} \to 0}$ ∴ $f(x) \to 5$

∴ range of $f(x)$ is $f(x) > 5$

∴ $f^{-1}(x) = \frac{4x+3}{5-x}$ $x > 5$

c) (i) $ff(x) = f\left(\frac{5x-3}{x-4}\right)$

$$= \frac{5\left(\frac{5x-3}{x-4}\right) - 3}{\left(\frac{5x-3}{x-4}\right) - 4} = \frac{25x - 15 - 3(x-4)}{5x - 3 - 4(x-4)}$$

Question 6 continued

$$= \frac{22x - 3}{x + 13}$$

(ii) we know $x > 4$

when $x = 4$ $\rightarrow$ $f(f(x)) = 5$ $\therefore$ we know $f(f(x)) > 5$

when $x \rightarrow \infty$ $\rightarrow$ $\dfrac{22 - \cancel{\frac{3}{x}}^{\nearrow 0}}{1 + \cancel{\frac{13}{x}}^{\nearrow 0}}$ $\therefore$ $f(f(x)) \rightarrow 22$

so range : $5 < f(f(x)) < 22$

7.

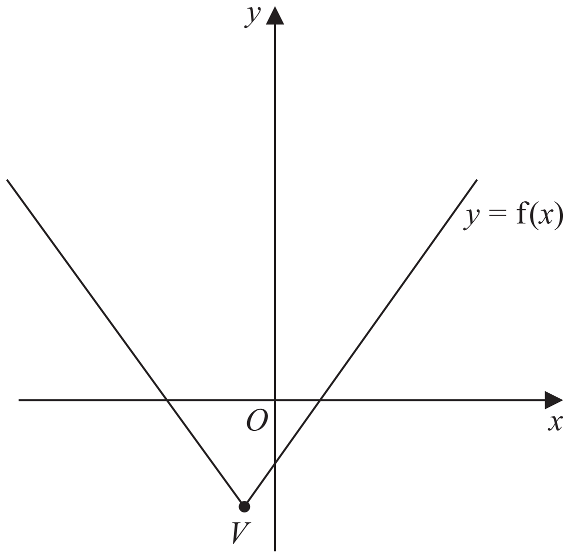


Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{1}{2}|2x + 7| - 10$$

(a) State the coordinates of the vertex, V, of the graph.
(2)

(b) Solve, using algebra,

$$\frac{1}{2}|2x + 7| - 10 \geqslant \frac{1}{3}x + 1$$

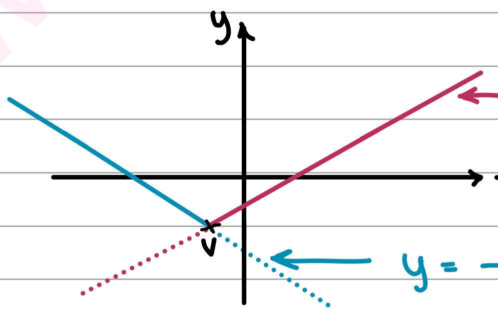
(4)

(c) Sketch the graph with equation

$$y = |\mathrm{f}(x)|$$

stating the coordinates of the local maximum point and each local minimum point.
(4)

7. a) $f(x) = 2|2x-5| + 3 \quad x \geq 0$



$y = \frac{1}{2}(2x+7) - 10 = x - \frac{13}{2}$

$y = -\frac{1}{2}(2x+7) - 10 = -x - \frac{27}{2}$

Question 7 continued

V is the point at which the 2 lines meet

$$x - \frac{13}{2} = -x - \frac{27}{2}$$

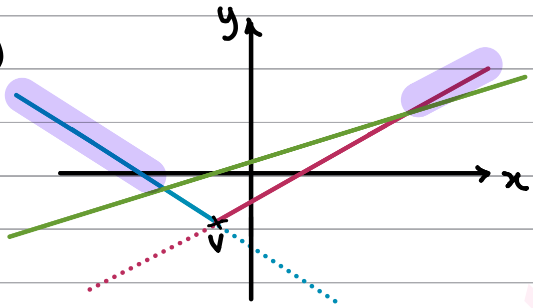
$$2x = -7$$

$$x = -\frac{7}{2}$$

$\therefore$ coordinates of P are

$$\left(-\frac{7}{2}, -10\right)$$

b)



$$f(x) \geqslant \frac{1}{3}x + 1$$

the line $y = \frac{1}{3}x + 1$ intersects with $f(x)$ twice

$$\frac{1}{3}x + 1 = -x - \frac{27}{2} \qquad\qquad \frac{1}{3}x + 1 = x - \frac{13}{2}$$

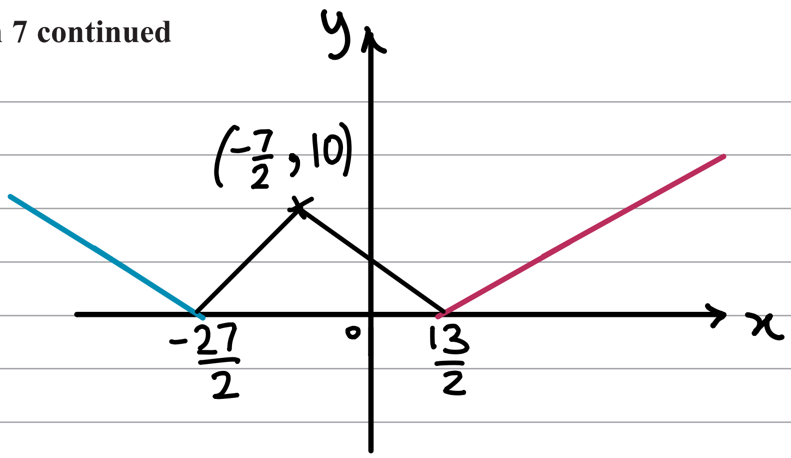
$$x = -\frac{87}{8} \qquad\qquad x = \frac{45}{4}$$

$$\therefore \quad x \leq -\frac{87}{8} \quad \cup \quad x \geqslant \frac{45}{4}$$

c) $y = |f(x)|$ $\leftarrow$ means all parts of graph are above x axis

Question 7 continued

∴



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8. A dose of antibiotics is given to a patient.

The amount of the antibiotic, x milligrams, in the patient's bloodstream t hours after the dose was given, is found to satisfy the equation

$$\log_{10} x = 2.74 - 0.079t$$

(a) Show that this equation can be written in the form

$$x = pq^{-t}$$

where p and q are constants to be found. Give the value of p to the nearest whole number and the value of q to 2 significant figures.

(4)

(b) With reference to the equation in part (a), interpret the value of the constant p.

(1)

When a different dose of the antibiotic is given to another patient, the values of x and t satisfy the equation

$$x = 400 \times 1.4^{-t}$$

(c) Use calculus to find, to 2 significant figures, the value of $\dfrac{dx}{dt}$ when $t = 5$

(3)

8. a) $\log_{10} x = 2.74 - 0.079t$

$\boxed{\text{LOG RULES} \rightarrow \log_a b = c \rightarrow a^c = b}$

$x = 10^{2.74 - 0.079t}$

$x = 10^{2.74} \times (10^{0.079})^{-t}$

$\therefore p = 10^{2.74} = 550 \qquad q = 10^{0.079} = 1.2$

b) when $t = 0 \rightarrow x = pq^{-0} = p$

$\therefore$ p is amount of antibiotic in patient's blood at the start

Question 8 continued

c) $x = 400 \times 1.4^{-t}$

$y = a^x$ ← rewrite a^x in terms of e

$a = e^{\ln(a)}$ $\frac{d}{dx}(e^{kx}) = ke^{kx}$

$\therefore y = (e^{\ln(a)})^x \rightarrow \frac{dy}{dx} = \ln(a) \times e^{(\ln a)x} = \ln(a) \times a^x$

$\therefore \frac{dA}{dt} = 400 \times -\ln(1.4) \times 1.4^{-t}$

↳ when $t = 5 \rightarrow \frac{dA}{dt} = -400 \ln(1.4) \times 1.4^{-5}$

$= -25$

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$2\sec^2 x - 3\tan x = 2$$

giving the answers, as appropriate, to 3 significant figures.

(4)

(ii) Prove that

$$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} \equiv 2$$

(4)

9.(i) $2\sec^2(x) - 3\tan(x) = 2$

$\sin^2 A + \cos^2 A = 1$ ← divide through by $\cos^2 A$

$\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A}$

$\boxed{\tan^2 A + 1 = \sec^2 A}$

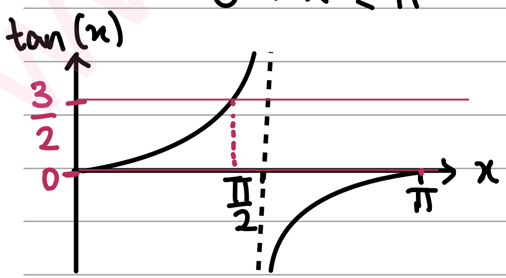
$2(\tan^2(x)+1) - 3\tan(x) = 2$

$2\tan^2(x) - 3\tan(x) = 0$

$\tan(x)(2\tan(x) - 3) = 0$

$\therefore \tan(x) = 0 \quad \cup \quad \tan(x) = \frac{3}{2}$

$0 < x \leq \pi$



$\therefore x = 0.983 \cup \pi$

Question 9 continued

(ii) LHS $= \frac{\sin(3\theta)}{\sin(\theta)} - \frac{\cos(3\theta)}{\cos(\theta)}$ RHS $= 2$

$= \frac{\sin(3\theta)\cos(\theta) - \cos(3\theta)\sin(\theta)}{\sin(\theta)\cos(\theta)}$

$\boxed{\text{USING COMPOUND ANGLE FORMULAE} \rightarrow \sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)}$

$= \frac{\sin(3\theta - \theta)}{\sin(\theta)\cos(\theta)}$ DOUBLE ANGLE FORMULAE $\sin(2A) = 2\sin(A)\cos(A)$

$= \frac{\cancel{\sin(2\theta)}}{\frac{1}{2}\cancel{\sin(2\theta)}}$

$= 2 = \text{RHS}$

10.

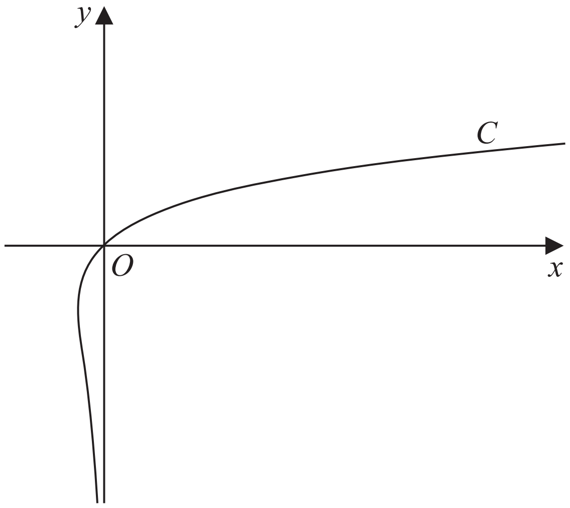


Figure 3

Figure 3 shows a sketch of the curve C with equation

$$x = ye^{2y} \qquad y \in \mathbb{R}$$

(a) Show that

$$\frac{dy}{dx} = \frac{y}{x(1+2y)}$$

(4)

Given that the straight line with equation $x = k$, where k is a constant, cuts C at exactly two points,

(b) find the range of possible values for k.

(3)

10. a) $x = ye^{2y}$

$\boxed{\text{PRODUCT RULE : } y = uv \quad y' = u'v + uv'}$

$u = y \qquad \frac{du}{dy} = 1$

$v = e^{2y} \qquad \frac{dv}{dy} = 2e^{2y}$

$\frac{dx}{dy} = 1(e^{2y}) + y(2e^{2y}) = e^{2y}(2y+1)$

P 6 9 3 1 1 A 0 3 0 3 2

Question 10 continued

$$\boxed{\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}}$$

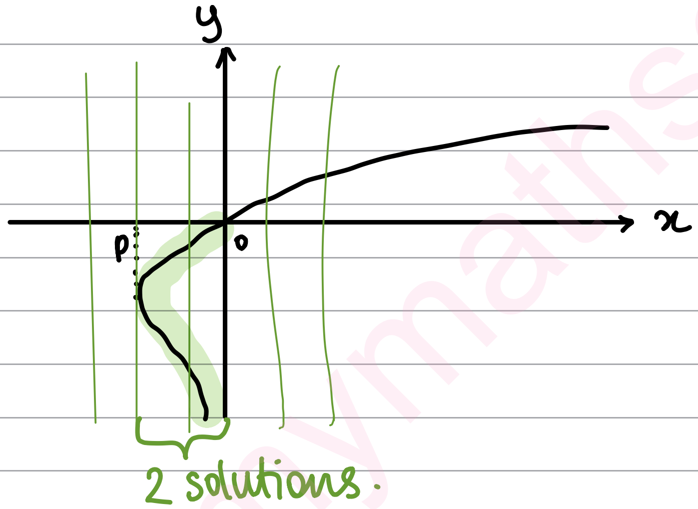
$$\therefore \frac{dy}{dx} = \frac{1}{e^{2y}(2y+1)}$$

$$= \frac{1}{2ye^{2y} + e^{2y}}$$

$$= \frac{1}{2x + \frac{x}{y}}$$

$$= \frac{y}{2xy + x} = \frac{y}{x(2y+1)}$$

b)



$x = k$ has 2 solutions when

$p < k < 0$

Stationary point p occurs when $\frac{dx}{dy} = 0$

$$e^{2y}(2y+1) = 0$$

$$y = -\frac{1}{2} \rightarrow \therefore p = -\frac{1}{2e}$$

$$\therefore -\frac{1}{2e} < k < 0$$